

Coherence depletion in the Grover quantum search algorithm

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We investigate the role of coherence in the Grover search algorithm by using several typical measures of quantum coherence and quantum correlations. By using the relative entropy of coherence measure, we show that the success probability depends on the depletion of coherence. Explicitly, in the limit case of few searcher items $j/N \ll 1$ in large database $N \gg 1$, the cost performance about coherence in enhancing the success probability of Grover search is related to the ratio j/N . The same phenomenon can also be found by using the l_1 norm of coherence, cost performance of coherence is inversely proportional to the scale of database N . In comparison, the behavior of quantum correlation measures, such as entanglement and discord, generally start from zero then reach the maximum and decrease to almost zero in the whole process of Grover search, the optimal success probability cannot be directly related with quantification of quantum correlation. Additionally, we find that quantum nonlocality does not appear in the Grover search since any two-qubit state does not violate CHSH type Bell inequality. Therefore, it is believed that the coherence can be viewed as a key resource for increasing the success probability in Grover search algorithm.

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I. INTRODUCTION

Quantum mechanics provides some distinctive computational resources that can be utilized to make quantum algorithms superior to any classical algorithms [1]. The origin of this speed-up in quantum computational processes has attracted many research attentions [2–5]. It is believed that entanglement plays a central role due to its wide applications in quantum information processes, such as Shor algorithm, quantum teleportation, superdense coding, quantum cryptography, etc [7–11, 23]. Besides entanglement, quantum discord, as another type of quantum correlations, is equally vital in quantum algorithms. For instance, discord is responsible for speed-up in deterministic quantum computation with one qubit (DQC1) [12, 13].

Coherence, a fundamental quantum property from the quantum pure states superposition principle [14], has been widely studied in various quantum information processing [15–17]. A rigorous framework for quantifying the coherence was proposed by Baumgratz *et al.* in Ref. [18]. Recently, coherence has been proved that it can be converted to other valued quantum resources, such as entanglement and discord, by suitable operations [19–21]. To some extent, coherence can be viewed as a more vital and fundamental quantum resource than entanglement. Moreover, coherence also exists in single systems without any correlation. A natural question is what the role of coherence plays in quantum algorithms?

At the heart of quantum algorithms, there lie a fundamental algorithms, Grover search algorithm [22, 23].

Grover search algorithm was introduced for accelerating search process [24]. It is believed that multi-partite entanglement is necessary for Grover search algorithm to achieve the speed-up [7]. To investigated properties of entanglement, different measures of entanglement, such as concurrence and geometric measure of entanglement, have been attempted in Grover search algorithm [25–30]. However, the role of entanglement is not yet fully demonstrated in the Grover search, in particular, the quantity of entanglement is not directly related with the success probability in the search [31]. On the other hand, quantum discord, as a nonclassical correlation beyond entanglement, has been proved that its behavior is not outstanding as well in Grover search [32]. It is worth noting that coherence is potentially a more fundamental quantum resource than entanglement or discord [33]. Much attention has been paid in this direction [34–39]. Maybe coherence takes responsibility for speed-up rather than entanglement or discord in Grover search. To clarify the role of coherence, we investigate coherence dynamics in Grover search. Other quantum correlations are also discussed for comparison.

This paper is organized as follows. In Sec. II, we briefly review Grover search algorithm and study its coherence dynamics of the whole n -qubit system in the cases of any solutions to the search problem by using two different coherence measures, namely, the relative entropy and the l_1 norm. In Sec. III, we consider entanglement, discord and nonlocality for any two qubits in the simplest situation of single solution to Grover search. Moreover, the multipartite entanglement for n -qubit system are also discussed. Finally, the main results are summarized in Sec. IV.

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II. COHERENCE DYNAMICS IN GROVER SEARCH ALGORITHM

The first step of Grover search algorithm is to initialize the n -qubit database to an equally weighted superposition of all computational basis states $|\psi_0\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle$, which can be realized by projecting prepared pure state $|0, \dots, 0\rangle$ to local Hadamard gates $H^{\otimes n}$ where $H = (|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)/\sqrt{2}$. It should be pointed that the initialized n -qubit database is a maximally coherent state with $N = 2^n$ equiprobable items $|x\rangle$ and our ultimate goal is to obtain desired items (in the following we call them as the “solutions” to Grover search algorithm) from it with the maximum probability after Grover search algorithm. The initialized database can be written in a more convenient form

$$|\psi_0\rangle = \sqrt{\frac{j}{N}} |X\rangle + \sqrt{\frac{N-j}{N}} |X^\perp\rangle, \quad (1)$$

where j represents the number of solutions and $|X\rangle = \frac{1}{\sqrt{j}} \sum_{x_s} |x_s\rangle$ ($|X^\perp\rangle = \frac{1}{\sqrt{N-j}} \sum_{x_n} |x_n\rangle$) is constructed by states $|x_s\rangle$ ($|x_n\rangle$) that are solutions (not solutions) to Grover search algorithm. It is easy to confirm that $|X\rangle$ and $|X^\perp\rangle$ are orthonormal. The next step is to apply Grover operation G repeatedly (called iteration) to improve proportion of solutions gradually. The Grover operation, $G = DO$, is comprised of oracle $O = \mathbf{1} - 2|X\rangle\langle X|$ and an inversion about average operation $D = 2|\psi_0\rangle\langle\psi_0| - \mathbf{1}$ [22]. After r iterations of the Grover operation G , the global state has the following form [1, 32]

$$|\psi_r\rangle \equiv G^r |\psi_0\rangle = \sin \alpha_r |X\rangle + \cos \alpha_r |X^\perp\rangle, \quad (2)$$

with $\alpha_r = (r+1/2)\alpha$ and $\alpha = 2 \arctan \sqrt{\frac{j}{N-j}}$. Note that $|\psi_r\rangle$ is also a pure state since G is unitary and initial state $|\psi_0\rangle$ is a pure state. The above processes are summarized in Fig. 1.

(1) Initialize the n -qubit database to $|\psi_0\rangle$;
 (2) Oracle O reflects the vector $|\psi_0\rangle$ according to $|X^\perp\rangle$ and then operation D reflects the vector $O|\psi_0\rangle$ according to $|\psi_0\rangle$. Therefore, the role of Grover operation G is to rotate the vector before iteration clockwise by an angle α . The final step is to measure $|\psi_r\rangle$ to obtain $|X\rangle$ with a high success probability. The success probability is described as

$$P(r) = \sin^2 \alpha_r. \quad (3)$$

Therefore, the wisest time to stop iteration is repeating times $r_{opt} = CI[\frac{\pi-\alpha}{2\alpha}]$ where $CI[x]$ denotes the closest integer to x . In the following, we restrict our discussion to range $0 \leq r \leq r_{opt}$.

Coherence, as a fundamental quantum correlation, describes the capability of a quantum state to exhibit quantum interference phenomena. The first rigorous framework to quantify the coherence was built by Baumgratz *et al.* in Ref. [18]. Based on this work, a number of coherence measures, such as the relative entropy of coherence,

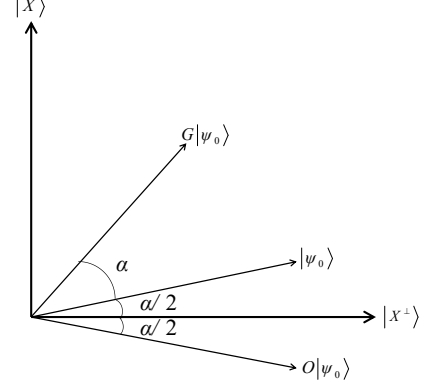


Figure 1: An illustration to show that the first two steps of the Grover search. Firstly, initialize the n -qubit database to $|\psi_0\rangle$; Secondly, O reflects $|\psi_0\rangle$ according to $|X^\perp\rangle$ and D reflects $O|\psi_0\rangle$ according to $|\psi_0\rangle$. Therefore, one whole iteration G turns the vector before iteration clockwise by an angle α .

the l_1 norm of coherence, the Tsallis relative α entropy of coherence and the coherence of formation [18, 40, 41], have been proposed. Recently, a novel phenomenon has been founded that all measures of coherence are frozen for an initial state in a strictly incoherent channel if and only if the relative entropy of coherence is frozen for the state in Ref. [42]. It means that the relative entropy of coherence maybe more vital than other coherence measures. Hence we choose the relative entropy as coherence measure and also calculate the l_1 norm of coherence for comparison. In this section, we consider coherence under the general case of j solutions. According to Eq. (2), the density matrix of state generated by Grover search can be written as

$$\begin{aligned} \rho(r) = & \frac{a^2}{j} \sum_{x_s, y_s} |x_s\rangle\langle y_s| + b^2 \sum_{x_n, y_n} |x_n\rangle\langle y_n| \\ & + \frac{ab}{\sqrt{j}} \left[\sum_{x_s} \sum_{y_n} (|x_s\rangle\langle y_n| + |y_n\rangle\langle x_s|) \right], \end{aligned} \quad (4)$$

where subscripts s and n denote that they are the solutions and not solutions, respectively. Here $a = \sin \alpha_r$ and $b = \frac{1}{\sqrt{N-j}} \cos \alpha_r$ are brought in for convenience.

A. The relative entropy of coherence

The definition of relative entropy of coherence is [18]

$$\mathcal{C}_r(\rho) = \min_{\delta \in \mathcal{I}} S(\rho \| \sigma), \quad (5)$$

where $S(\rho \| \sigma) = \text{Tr}(\rho \log_2 \rho - \rho \log_2 \sigma)$ is the quantum relative entropy and \mathcal{I} denotes a set of incoherent quantum states whose density matrices are diagonal in the calculational basis. This formula can be written as a closed

form [18], avoiding the minimization

$$\mathcal{C}_r(\rho) = S(\rho_{diag}) - S(\rho), \quad (6)$$

where $\rho_{diag} = \sum_i \rho_{i,i} |i\rangle\langle i|$ and $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy.

Substitute the Eq. (4) into Eq. (6), we obtain the coherence dynamics of n -qubit

$$\mathcal{C}_r(\rho) = H(a^2) + \log_2(N-j) + a^2 \log_2 \frac{j}{N-j}, \quad (7)$$

where $H(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ is the binary Shannon entropy function. Note that the relative entropy of coherence is independent of the choices of solutions. In other words, it only depends on the number of solutions j since $S(\rho) = 0$ ($\rho(r)$ is a pure state) and $S(\rho_{diag})$ is only connected with the diagonal elements of $\rho(r)$. From Eq. (7), we have

$$\frac{d\mathcal{C}_r(\rho)}{dr} = \log_2 \frac{j(1-a^2)}{(N-j)a^2} \sin(2\alpha_r) \alpha \leq 0 \quad (8)$$

for $0 \leq r \leq r_{opt}$ due to $a(r) = \sin \alpha_r \geq a(0) = \sqrt{\frac{j}{N}}$, which means that $\mathcal{C}_r(\rho)$ is a decreasing function of r . On the contrary, the success probability $P(r)$ is a increasing function for $0 \leq r \leq r_{opt}$. Moreover, the coherence achieves the minimal value while the probability of success reaches the maximal value 1. That is to say, the coherence can be viewed as a key resource in Grover search which improves the success probability by consuming itself, see Fig. 2.

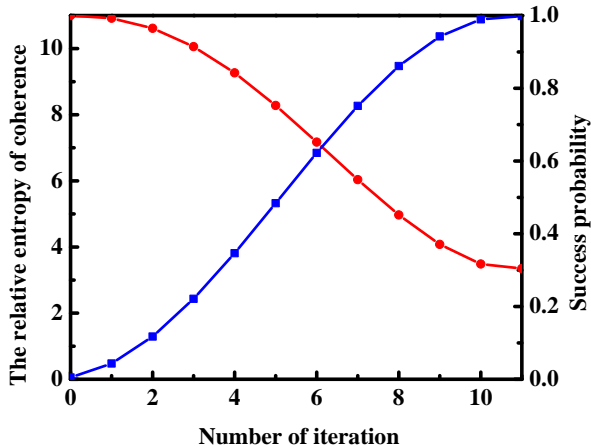


Figure 2: (Color online) The relative entropy of coherence for the whole 11-qubit system with $j = 10$ solutions. The coherence is depicted by red dots and the success probability by blue squares.

It is possible to express the coherence $\mathcal{C}_r(\rho)$ as a function of the success probability P . Due to the fact that $P = a^2$, the coherence is rewritten as

$$\mathcal{C}_r(\rho) = H(P) + \log_2(N-j) + P \log_2 \frac{j}{N-j}. \quad (9)$$

Actually, the Grover search algorithm is usually applied in the situation that a few solutions in a huge database. Under this condition ($j \ll N$ and $N \gg 1$), $H(P)$ can be omitted compared with $\log_2(N-j)$ and Eq. (9) takes the following form

$$\mathcal{C}_r(\rho) \simeq -P \log_2 \frac{N}{j} + \log_2 N, \quad (10)$$

which is a linear function of P . Above formula shows that the depletion of coherence $\mathcal{C}_r(\rho)$ leads to improving the success probability P . This ability can be quantified as cost performance w ,

$$w = -\frac{\Delta P}{\Delta \mathcal{C}_r} = \frac{1}{\log_2 \frac{N}{j}}. \quad (11)$$

Clearly, the cost performance is related with a constant $\frac{j}{N}$, which represent the ratio of number of solutions to the scale of database.

B. The l_1 norm of coherence

The l_1 norm of coherence is a very intuitive quantification which comes from a simple fact that coherence is linked with the off-diagonal elements of considered quantum states. The expression of the l_1 norm of coherence is expressed as [18]

$$\mathcal{C}_{l_1}(\rho) = \sum_{i \neq j} |\rho_{i,j}|. \quad (12)$$

Employing this equation, we have the coherence dynamics in the Grover algorithm

$$\mathcal{C}_{l_1}(\rho) = (\sqrt{j} \sin \alpha_r + \sqrt{N-j} \cos \alpha_r)^2 - 1, \quad (13)$$

when $0 \leq r \leq r_{opt}$. Using Eq. (3), the l_1 norm of coherence can be rewritten as the function of P

$$\mathcal{C}_{l_1}(\rho) = (\sqrt{jP} + \sqrt{(N-j)(1-P)})^2 - 1. \quad (14)$$

In the asymptotic limits $j \ll N$ and $N \gg 1$, the l_1 norm of coherence takes the simple form

$$\mathcal{C}_{l_1}(\rho) \simeq -NP + N, \quad (15)$$

which means that the same phenomenon that improving the success probability by cost of coherence is also existed under the l_1 norm measure of coherence. The cost performance w equals to $1/N$. For this reason, we demonstrate that coherence is a key quantum resource to enhance the success probability in Grover search.

III. OTHER QUANTUM CORRELATIONS IN GROVER SEARCH ALGORITHM

In this section, we only consider the simplest situation of single solution ($j = 1$). Without loss of generality,

assume that the solution located at $|0\rangle$ and the density matrix of states generated by Grover search (Eq. (4)) has the following form

$$\rho(r) = \begin{pmatrix} a^2 & ab & ab & ab & \cdots \\ ab & b^2 & b^2 & b^2 & \cdots \\ ab & b^2 & b^2 & b^2 & \cdots \\ ab & b^2 & b^2 & b^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}_{N \times N}. \quad (16)$$

A. Entanglement in Grover search

Entanglement, a quantum resource, is widely considered as the main undertaker for quantum computational speed-up though the role of entanglement is not clear. Here we use concurrence, a widely-accepted entanglement measure, to investigate the behavior of entanglement during the Grover search algorithm and compare it with coherence. The concurrence of arbitrary two-qubit states is defined in Ref [43] and is calculated as follows

$$E_2(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (17)$$

where λ_i s are square roots of the eigenvalues of matrix $\rho\tilde{\rho}$ in decreasing order, $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$. Here $\tilde{\rho} =$

$(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, where σ_y is Pauli matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and ρ^* is the conjugation of ρ . According to Eq. (17) and Eq. (20), the expression of concurrence between any two qubits ρ_2 in Grover search can be obtained [25]

$$E_2(\rho_2) = 2|B - C| = 2|ab - b^2|. \quad (18)$$

The behavior of pairwise entanglement in the case of $n = 11$ is displayed in Fig. 3. The pairwise entanglement firstly increases to the maximal value and then decreases to almost zero when the optimal number of iterations is reached.

Now let's consider the multipartite entanglement of n -qubit system, which may better depict the behavior of $P(r)$. The concurrence of n -qubit states is induced in Ref [44]

$$E_n(\psi) = \frac{2}{\sqrt{N}} \sqrt{(N-2)\langle\psi|\psi\rangle^2 - \sum_{\beta} \text{Tr}\rho_{\beta}^2}, \quad (19)$$

where $N = 2^n$ and β labels $(N-2)$ different reduced density matrices; i.e., there are C_N^k different terms when tracing over k different subsystems from the n -qubit system. From Eq. (16), we have reduced matrix for any k -qubit

$$\rho_k = \begin{pmatrix} a^2 + (2^{n-k} - 1)b^2 & ab + (2^{n-k} - 1)b^2 & ab + (2^{n-k} - 1)b^2 & ab + (2^{n-k} - 1)b^2 & \cdots \\ ab + (2^{n-k} - 1)b^2 & 2^{n-k}b^2 & 2^{n-k}b^2 & 2^{n-k}b^2 & \cdots \\ ab + (2^{n-k} - 1)b^2 & 2^{n-k}b^2 & 2^{n-k}b^2 & 2^{n-k}b^2 & \cdots \\ ab + (2^{n-k} - 1)b^2 & 2^{n-k}b^2 & 2^{n-k}b^2 & 2^{n-k}b^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}_{2^k \times 2^k}. \quad (20)$$

Thereby the concurrence of whole n -qubit system can be expressed as

$$E_n = \frac{2}{\sqrt{N}} \sqrt{(N-2) - \sum_{k=1}^{n-1} C_n^k \text{Tr}\rho_k^2}. \quad (21)$$

Substitute Eq. (20) into the above equation, we have

$$\begin{aligned} E_n = & \frac{2}{\sqrt{2^n}} [2^n - 2 - (4 \times 3^n - 2^{n+3} + 4)a^2b^2 \\ & - (8^n + 4 \times 3^n - 3 \times 2^{2n+1} + 3 \times 2^n - 2)b^4 \\ & - (2^n - 2)a^4 - 4(4^n - 2 \times 3^n + 2^n)ab^3]^{\frac{1}{2}}. \end{aligned} \quad (22)$$

Using this equation, we present the behavior of multipartite entanglement of n -qubit system in the case that $n = 11$, which is similar with the pairwise entanglement (see Fig. 3). This result implies that the entanglement, pairwise entanglement or multipartite entan-

glement, cannot provide us more information than coherence in Grover search.

B. Discord in Grover search

Discord was introduced in Ref. [45] to quantify quantum correlation, which is viewed as the difference between the total correlation and classical correlation

$$\mathcal{D}(\rho) = \mathcal{I}(\rho) - \mathcal{C}(\rho), \quad (23)$$

where \mathcal{I} and \mathcal{C} represent the total correlation and classical correlation, respectively. In Ref. [46], the total correlation between two systems A and B are defined by the minimal amount of noise needed to destroy all the correlation between them, which is equal to the quantum mutual information

$$\mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \quad (24)$$

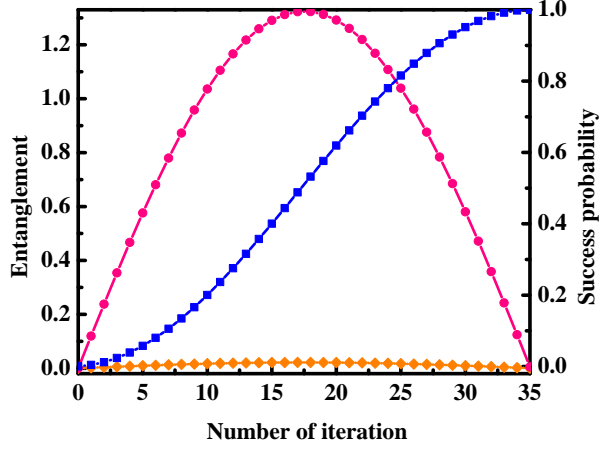


Figure 3: (Color online) The evolutions for the entanglement in the case of 11-qubit system. The pairwise entanglement is depicted by orange diamonds while the entanglement of the whole 11-qubit system by pink points. The blue squares represent the success probability.

where $\rho_{A(B)} = \text{Tr}_{B(A)} \rho_{AB}$. The classical correlation was proposed in Ref. [47] as the maximum information we can obtain from A by measuring B . Under projective measurements $\{\Pi_i\}$, the classical correlation can be written as

$$\mathcal{C}(\rho) = \max_{\{\Pi_i\}} \{S(\rho_A) - \sum_i p_i S(\rho_{A|i})\}, \quad (25)$$

where $p_i = \text{Tr}_{AB}(I \otimes \Pi_i) \rho_{AB} (I \otimes \Pi_i)$ and $\rho_{A|i} = \frac{1}{p_i} \text{Tr}_B(I \otimes \Pi_i) \rho_{AB} (I \otimes \Pi_i)$. Thus, discord is written as

$$\mathcal{D}(\rho) = \min_{\{\Pi_i\}} \sum_i [p_i S(\rho_{A|i}) + S(\rho_B) - S(\rho_{AB})]. \quad (26)$$

We choose the two-qubit discord to analyse discord behavior in the Grover search. The projective measurement can be parameterized via $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$ in the form of $\{\cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle, e^{-i\phi} \sin \theta |0\rangle - \cos \theta |1\rangle\}$. Using the numerical method, we calculate pairwise discord in the case of 11-qubit system. The Fig. 4 shows that the behavior of pairwise discord is similar to the entanglement.

C. Nonlocality in Grover search

Nonlocality is another manifestation of nonclassical correlation which tells us that reproducing the predictions of quantum theory by considering local hidden variables (LHV) is impossible. It is well known that the entanglement is necessary for the existence of nonlocality but nonlocality is not necessary for entanglement [48]. We are interested about whether nonlocality produces speed-up in Grover search or not. Unfortunately, there is a lack of necessary and sufficient criterions or suitable measurements for nonlocality. Violating the CHSH

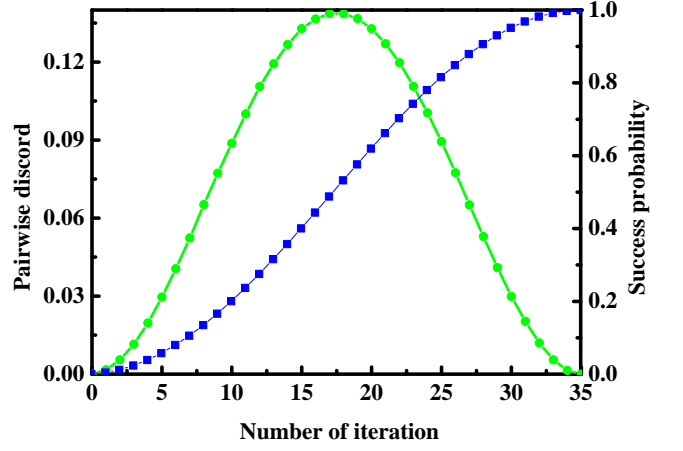


Figure 4: (Color online) The evolutions for the pairwise discord in the case of 11-qubit system. The pairwise discord is depicted by green points while the success probability by blue squares.

inequality provides a powerful tool to recognize the nonlocality of two-qubit systems. Consequently, we choose the CHSH inequality to investigate the nonlocality of any two qubits during the Grover search.

The reduced matrix of any two qubits takes the following form

$$\rho_2 = \begin{pmatrix} A & B & B & B \\ B & C & C & C \\ B & C & C & C \\ B & C & C & C \end{pmatrix}, \quad (27)$$

where $A = a^2 + (\frac{N}{4} - 1)b^2$, $B = ab + (\frac{N}{4} - 1)b^2$ and $C = \frac{N}{4}b^2$. In Ref. [49], a theorem, that a two-qubit system violates the CHSH inequality if and only if $M(\rho) > 1$, has been given. Note that obeying the CHSH inequality does not mean that the system is local. Here, $M(\rho) = \max_{i \neq j} \{u_i + u_j\}$ with u_i being the three eigenvalues of the matrix $T^T T$, where $T = T_{ij} = \text{Tr} \rho(\sigma_i \otimes \sigma_j)$ is the correlation matrix. The correlation matrix for ρ_2 is given by

$$T = \begin{pmatrix} 2B + 2C & 0 & 2B - 2C \\ 0 & 2C - 2B & 0 \\ 2B - 2C & 0 & A - C \end{pmatrix} \quad (28)$$

and the corresponding eigenvalues are $\lambda_1 = 2C - 2B$, $\lambda_2 = \frac{(A+2B+C)-\sqrt{\Delta}}{2}$ and $\lambda_3 = \frac{(A+2B+C)+\sqrt{\Delta}}{2}$ with $\Delta = A^2 + 20B^2 + 25C^2 - 4AB - 6AC - 20BC$. Therefore, we have

$$M(\rho_2) = \begin{cases} \lambda_2^2 + \lambda_3^2, & \lambda_1 \leq \lambda_2; \\ \lambda_1^2 + \lambda_3^2, & \lambda_1 > \lambda_2. \end{cases} \quad (29)$$

In the asymptotic limits $N \gg 1$, we have $\lambda_1 \leq \lambda_2$ and

$$\begin{aligned}
 M(\rho_2) &= \lim_{N \rightarrow \infty} \lambda_2^2 + \lambda_3^2 = \lim_{N \rightarrow \infty} \frac{(A + 2B + C)^2 + \Delta}{2} \\
 &= \lim_{N \rightarrow \infty} A^2 + 12B^2 + 13C^2 - 2AC - 8BC \\
 &= 1 - 2\sin^2\left(\frac{2r+1}{2}\alpha\right)\cos^2\left(\frac{2r+1}{2}\alpha\right) \\
 &\leq 1,
 \end{aligned} \tag{30}$$

which means that the pairwise nonlocality does not exist in this limit case.

IV. CONCLUSIONS

In this work, we have systematically studied the evolutions of coherence and other typical quantum correlations in the process of the standard Grover search. By using the relative entropy measure of coherence, we show that the improvement of success probability relies on the depletion of coherence for any number of solutions. Explicitly, in the limit case of a few searcher items $j \ll N$ and large database $N \geq 1$, the cost performance about coherence in enhancement the success probability is related to the ratio of number of searched solutions to the scale of database, j/N . The same phenomenon also exists by using the l_1 norm of coherence and corresponding cost performance equals to $1/N$. Consequently, coher-

ence can be viewed as a key resource for increasing the success probability in Grover search algorithm.

The multipartite entanglement of n -qubit fails to describe variation of the success probability more meaningful than pairwise entanglement, calculated in Ref. [25]. The pairwise discord can not be directly connected with success probability either in Grover search. Moreover, in the limit case, the nonlocality of any two-qubit systems does not appear since CHSH type Bell inequality is not violated in Grover search. In a word, the coherence is a key resource and tightly connected to the success probability in Grover search.

Our results contribute to the resource theory of quantum coherence and provide insights into the role of coherence in quantum algorithms. It would be interesting to study other quantum resources, such as steering, in quantum information process [48, 50].

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